Oxford Cambridge and RSA

Monday 18 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,dx = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

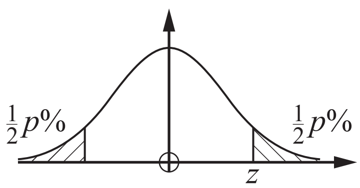
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 **(a)** Express x^2+8x+2 in the form $(x+a)^2+b$. **[2]**

(b) Write down the coordinates of the turning point of the curve $y=x^2+8x+2$. **[1]**

(c) State the transformation(s) which map(s) the curve $y=x^2$ onto the curve $y=x^2+8x+2$. **[2]**

2 Solve the equation $\sin 2x = 0.3$ for $0° \leqslant x \leqslant 180°$. Give your answer(s) correct to **1** decimal place. **[2]**

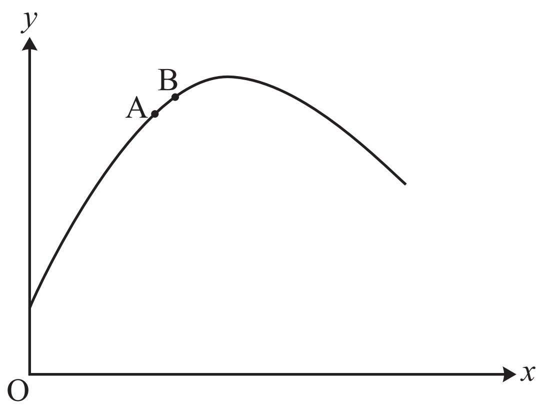
3 **(a)** Determine, in terms of k, the coordinates of the point where the lines with the following equations intersect.

$$x+y=k$$
$$2x-y=1$$ **[3]**

(b) Determine, in terms of k, the coordinates of the points where the line $x+y=k$ crosses the curve $y=x^2+k$. **[4]**

4 The diagram shows points A and B on the curve $y=\left(\frac{x}{4}\right)^{-x}$.

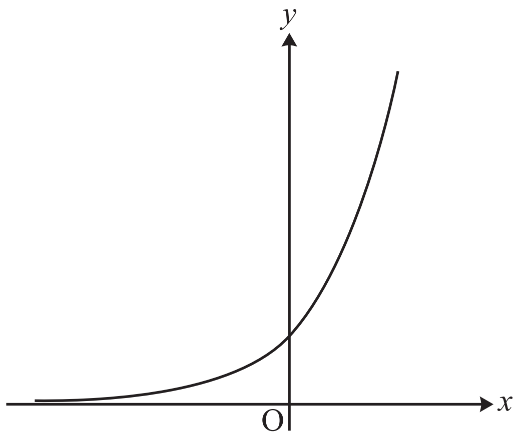
The x-coordinate of A is 1 and the x-coordinate of B is 1.1.



(a) Find the gradient of chord AB. Give your answer correct to **2** decimal places. **[2]**

(b) Give the x-coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. **[1]**

5 **(a)** The diagram shows the curve $y = e^x$.



On the axes in the Printed Answer Booklet, sketch graphs of

(i) $\dfrac{dy}{dx}$ against x, **[1]**

(ii) $\dfrac{dy}{dx}$ against y. **[2]**

(b) Wolves were introduced to Yellowstone National Park in 1995.

The population of wolves, y, is modelled by the equation

$y = Ae^{kt}$,

where A and k are constants and t is the number of years after 1995.

(i) Give a reason why this model might be suitable for the population of wolves. **[1]**

(ii) When $t = 0$, $y = 21$ and when $t = 1$, $y = 51$.

Find values of A and k consistent with the data. **[3]**

(iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. **[1]**

6 **In this question you must show detailed reasoning.**

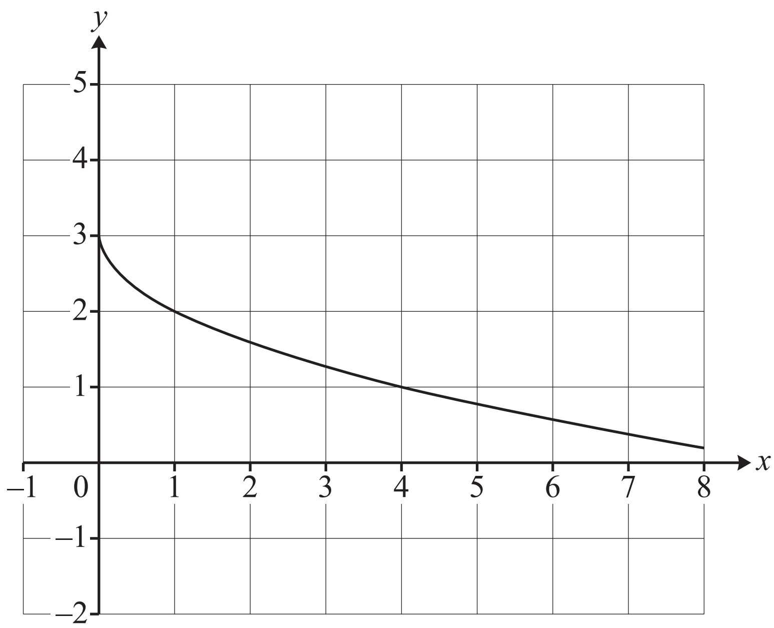
Show that $\sum_{r=1}^{3} \frac{1}{\sqrt{r+1}+\sqrt{r}} = 1$. **[4]**

7 Determine $\int x \cos 2x \, dx$. **[3]**

8 For a particular value of a, the curve $y = \frac{a}{x^2}$ passes through the point (3, 1).

Find the coordinates of all the other points on the curve where both the x-coordinate and the y-coordinate are integers. **[3]**

9 The diagram shows the curve $y = 3 - \sqrt{x}$.



(a) Draw the line $y = 5x - 1$ on the copy of the diagram in the Printed Answer Booklet. **[1]**

(b) **In this question you must show detailed reasoning.**

Determine the exact area of the region bounded by the curve $y = 3 - \sqrt{x}$, the lines $y = 5x - 1$ and $x = 4$ and the x-axis. **[10]**

10 **(a)** Express $\dfrac{1}{(4x+1)(x+1)}$ in partial fractions. **[3]**

(b) A curve passes through the point (0, 2) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{(4x+1)(x+1)},$$

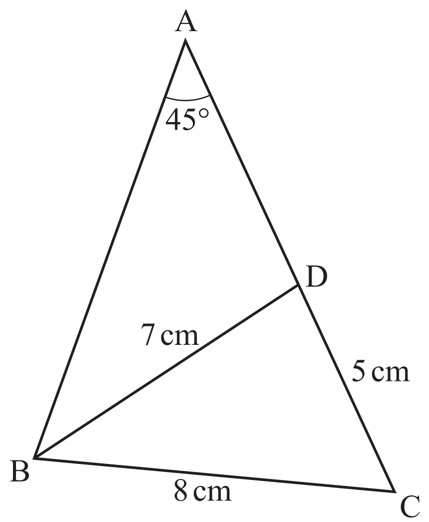
for $x > -\frac{1}{4}$.

Show by integration that $y = A\left(\dfrac{4x+1}{x+1}\right)^{B}$ where A and B are constants to be determined. **[6]**

11 **In this question you must show detailed reasoning.**

The diagram shows triangle ABC, with BC = 8 cm and angle BAC = 45°.

The point D on AC is such that DC = 5 cm and BD = 7 cm.



Determine the exact length of AB. **[5]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Show that $\beta = \arctan\left(\frac{1}{3}\right)$, as given in line 15. **[3]**

13 **(a)** Use triangle ABE in **Fig. C2** to show that $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$, as given in line 29. **[1]**

(b) Sketch the graph of $y = \arctan x$. **[1]**

(c) What property of the arctan function ensures that $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$, as given in line 30? **[1]**

14 **(a)** Show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right) \Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1.$$ **[1]**

(b) Use the arctan addition formula in line 23 to show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right),$$ as given in line 39. **[4]**

15 Prove that $\arctan 1 + \arctan 2 + \arctan 3 = \pi$, as given in line 41. **[4]**

END OF QUESTION PAPER